CSCI-400: Lambda Calculus Homework

Name: _____ CWID: _____

- A **printed** copy is due in class on Thursday, February 21.
- Handwritten or typed answers are acceptable. If handwritten, please write clear and legibly.

1. (50 points) Fully beta reduce each of the following lambda terms. Check your answer using the `lc` tool shown in class. **Do not use any shorthands in your evaluation**.

   Note: it is acceptable to skip currying steps. You should not skip any other steps: show all of you work!

   (a) $(\lambda p.\lambda q.pqp)(\lambda x.\lambda y.y)(\lambda x.\lambda y.x)$

   (b) $(\lambda p.\lambda q.ppq)(\lambda x.\lambda y.y)(\lambda x.\lambda y.x)$

   (c) $(\lambda p.p(\lambda x.\lambda y.y)(\lambda x.\lambda y.x))(\lambda x.\lambda y.y)$

(d) $(\lambda p.\lambda a.\lambda b.pab)(\lambda x.\lambda y.y)(\lambda f.(\lambda x.fx))(\lambda f.(\lambda x.f(fx)))$

(e) $(\lambda c.c(\lambda x.\lambda y.y))(\lambda x.\lambda y.\lambda f.fxy)(\lambda f.\lambda x.f(fx))(\lambda f.\lambda x.f(f(fx)))$

2. (20 points) Identify which variables are free and which are bound in each of the lambda terms:
   (a) $\lambda x.xy$

   (b) $(\lambda x.x)m$

3. (10 points (bonus)) Write a lambda calculus abstraction which, when a Church numeral n is applied, evaluates to the Church numeral $n - 1$. Note that this is the inverse of the `SUCC` function. Then, explain how your abstraction works and provide an example for the Church numeral 5. (Note that this abstraction need not handle 0.)