Haskell: Higher Order Functions (Part I)

Principles of Programming Languages

Colorado School of Mines

https://lambda.mines.edu

Review: Functions are Data

Haskell lets us pass functions as the arguments to other functions:

```haskell
double :: (Num a) => a -> a
double x = x * 2

doubleAll :: (Num a) => [a] -> [a]
doubleAll xs = map double xs
```

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And we can define functions which take functions:

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map' :: (a -> b) -> [a] -> [b]
map' _ [] = []
map' f (x:xs) = f x : map f xs
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We see Haskell treats functions as a **first-class citizen**; that is, we can pass them around just like any other type of data.

Review: Currying

- Haskell takes advantage of **currying** to support functions with multiple arguments. That is, functions take a single argument and return a function ready to take the next argument.
- We call the function ready to take the next argument a **partially applied function**.

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```
subtractMinutes :: Int -> Int -> Int
subtractMinutes n x = (x - n) `mod` 60

-- define a function which subtracts
-- 45 minutes every time
--      subtract45 30 --> 45
subtract45 = subtractMinutes 45
```

Partially Applied Prefix Functions

```haskell
multiplyBy :: (Num a) => a -> a -> a
multiplyBy x y = x * y

-- define our doubleAll using a partially applied
-- prefix function (and currying!)
doubleAll = map (multiplyBy 2)
```

Partially Applied Infix Functions

Write a partially complete infix function in parentheses to create a partially applied infix function.

```haskell
-- define our doubleAll using a partially applied
-- infix function (and currying!)
doubleAll = map (2 *)

-- also valid
doubleAll = map (* 2)
```

zipWith

`zipWith` is a really useful function in Haskell's standard library. It takes a function that takes two arguments, and applies it to two each of the elements from two lists. For example:

```
GHCi> zipWith (+) [1,2,3] [10,20,30]
[11,22,33]
GHCi> zipWith max [1..5] (reverse [1..5])
[5,4,3,4,5]
```

Let's Implement zipWith

```
zipWith' :: (a -> b -> c) -> [a] -> [b] -> [c]
zipWith' _ [] _ = []
zipWith' _ _ [] = []
zipWith' f (x:xs) (y:ys) = f x y : zipWith' f xs ys
```

Anonymous Functions (Lambdas)

Haskell provides a notation to write functions inline without a name:

```haskell
-- twistTuples [(1,2),(3,4)] --> [(2,1),(4,3)]
twistTuples xs = map (\ (a,b) -> (b,a)) xs
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Why do we have lambdas? Perhaps there is a case where writing a lambda might be cleaner than another function, a `let` or `where` binding, or partial application.

Maybe Lambda Makes This Cleaner

Perhaps a lambda can make it more clear we are returning another function. Consider the `flip` function (in Haskell's standard library) which takes a function and returns a new one with the arguments flipped:

```
flip' :: (a -> b -> c) -> b -> a -> c
flip' f x y = f y x
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```
flip' :: (a -> b -> c) -> b -> a -> c
flip' f x y = f y x
```

Is it immediately obvious this function is supposed to return another (partially applied) function? Compare to this definition:

```
flip' :: (a -> b -> c) -> b -> a -> c
flip' f = \ x y -> f y x
```

With your learning groups, everyone take turns taking your quizzes you designed. Once finished, we will start Quiz 2.

More on higher order functions next time.