

Haskell: Higher Order Functions (Part II)

Principles of Programming Languages

Colorado School of Mines

<https://lambda.mines.edu>

The \$ Function

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So it's still worthless, right? What if I told you that it has the lowest precedence and is right-associative?

Using \$ to reduce parentheses

Function application using spaces is left-associative and high precedence, so $f\ a\ b\ c$ is equivalent to $((f\ a)\ b)\ c$.

What if a and b were functions and we wanted $f\ (a\ (b\ c))$ instead? We had to add lots of parentheses and it gets messy fast.

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Let's use $\$$ to fix this:

-- *The following two expressions are equivalent*

$f\ (a\ (b\ c))$

$f\ \$\ a\ \$\ b\ c$

Reducing Parentheses: More Examples

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More examples:

- What does `sqrt 3 + 4 + 5` compute?
- What does `sqrt $ 3 + 4 + 5` compute?

More \$ Tricks: Partial Application

```
-- This function takes a list of functions and applies  
-- [1..10] to each  
onCountToTen = map ($) [1..10]  
  
-- For example:  
-- onCountToTen [filter even, filter odd, map (*2)]  
-- [[2,4,6,8,10],[1,3,5,7,9],[2,4,6,8,10,12,14,16,18,20]]
```

\$: What I expect you to know

- How to **intepret** an expression which uses \$
- How to **use** \$ to reduce parentheses
- How to **use** a partial application of \$ to apply an argument to a list of functions

\$: What I expect you to know

- How to **intepret** an expression which uses \$
- How to **use** \$ to reduce parentheses
- How to **use** a partial application of \$ to apply an argument to a list of functions

Understanding the definition of the \$ function and it's precedence is optional, but I think it's helpful to figure out the above.

Function Composition

In mathematics, if we have a function $f(x)$ and $g(x)$, we can rewrite $f(g(x))$ as:

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In Haskell, this \circ can equivalently be written as `.`:

```
sumOfSquares = sum . (^2)
```


Which do you choose?

-- All of these are equivalent, which would you write?

```
crazy x y = floor (negate (tan (sin (max x y))))
```

```
crazy x y = floor $ negate $ tan $ sin $ max x y
```

```
crazy = floor . negate . tan . sin . max
```

Reduction Functions

A **reduction function** is a function which takes a list and reduces the elements in the list to a single value. For example, `sum` and `product` are reduction functions:

```
GHCi> sum [1..10]
```

```
55
```

```
GHCi> product [1..10]
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3628800
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```

What if we had a generalized reduction function which took a function and applied it across a sequence to obtain a result? Something like this:

$$\text{reduce}(f, \text{seq}) = f(f(f(\text{seq}[0], \text{seq}[1]), \text{seq}[2]), \dots)$$

Fold Right

Haskell has a function called `foldr`, which takes a function, an initial value, and a list, and applies the function to each element in the list, recursively calling `fold` for the right value.

```
foldr f z [] = z
foldr f z (x:xs) = f x (foldr f z xs)
```

Fold Left

Haskell has a function called `foldl`, which takes a function, an initial value, and a list. It recurses immediately, making the new initial value the result of calling the function on the initial value and the current element.

```
foldl f z [] = z
foldl f z (x:xs) = foldl f (f z x) xs
```

Examples: Folding

```
-- sum using foldl  
sum' = foldl (+) 0
```

```
-- sum using foldr  
sum' = foldr (+) 0
```

```
-- product  
product' = foldl (*) 1
```

Quiz Prep Time

With your new learning groups, take some time preparing for the quiz using whatever study mechanism you wish.

Topics covered:

- Pattern Matching and Recursion
- Let, Where, Case, Guards