Haskell: Higher Order Functions (Part II)

Principles of Programming Languages

Colorado School of Mines

https://lambda.mines.edu
Haskell has an infix function: \$. Here is how it’s defined:

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\[ f \$ x = f x \]
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**What the heck is this worthless function?** It's a function applicator: it takes a function on the left and an argument on the right, and applies the function to the argument.
Haskell has an infix function: \$. Here is how it’s defined:

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\]

\[
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**What the heck is this worthless function?** It’s a function applicator: it takes a function on the left and an argument on the right, and applies the function to the argument.

**So it’s still worthless, right?** What if I told you that it has the lowest precedence and is right-associative?
Function application using spaces is left-associative and high precedence, so \( f \ a \ b \ c \) is equivalent to (((\( f \ a \) \ b) \ c)).

What if \( a \) and \( b \) were functions and we wanted \( f \ (a \ (b \ c)) \) instead? We had to add lots of parentheses and it gets messy fast.
Function application using spaces is left-associative and high precedence, so \( f \ a \ b \ c \) is equivalent to \(((f \ a) \ b) \ c\).

What if \( a \) and \( b \) were functions and we wanted \( f \ (a \ (b \ c)) \) instead? We had to add lots of parentheses and it gets messy fast.

Let’s use \$\$ to fix this:

```
-- The following two expressions are equivalent
    f (a (b c))
    f $ a $ b c
```
Reducing Parentheses: More Examples

- \texttt{length \ (filter \ odd \ [1..10])}
Reducing Parentheses: More Examples

- length (filter odd [1..10])
- length $ filter odd [1..10]
Reducing Parentheses: More Examples

- `length (filter odd [1..10])`
- `length $ filter odd [1..10]`

- `sum (map sqrt (filter even [1..100]))`
Reducing Parentheses: More Examples

- \( \text{length \ (filter \ odd \ [1..10]\)} \)
- \( \text{length \ $\ filter \ odd \ [1..10]\)} \)

- \( \text{sum \ (map \ sqrt \ (filter \ even \ [1..100]\))} \)
- \( \text{sum \ $\ map \ sqrt \ $\ filter \ even \ [1..100]\} \)

More examples:

- What does \( \text{sqrt \ 3 + 4 + 5} \) compute?
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Reducing Parentheses: More Examples

- `length (filter odd [1..10])`
- `length $ filter odd [1..10]`

- `sum (map sqrt (filter even [1..100]))`
- `sum $ map sqrt $ filter even [1..100]`

More examples:

- What does `sqrt 3 + 4 + 5` compute?
- What does `sqrt $ 3 + 4 + 5` compute?
More $ Tricks: Partial Application

-- This function takes a list of functions and applies
-- [1..10] to each
onCountToTen = map ($ [1..10])

-- For example:
-- onCountToTen [filter even, filter odd, map (*2)]
-- [[2,4,6,8,10],[1,3,5,7,9],[2,4,6,8,10,12,14,16,18,20]]
How to interpret an expression which uses $ 
How to use $ to reduce parentheses  
How to use a partial application of $ to apply an argument to a list of functions
$: What I expect you to know

- How to **interpret** an expression which uses $
- How to **use** $ to reduce parentheses
- How to **use** a partial application of $ to apply an argument to a list of functions

Understanding the definition of the $ function and it’s precedence is optional, but I think it’s helpful to figure out the above.
In mathematics, if we have a function $f(x)$ and $g(x)$, we can rewrite $f(g(x))$ as:

$$(f \circ g)(x)$$
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$$(f \circ g)(x)$$

In Haskell, this $\circ$ can equivalently be written as .:

```haskell
sumOfSquares = sum . (^2)
```
Which do you choose?

-- All of these are equivalent, which would you write?
crazy x y = floor (negate (tan (sin (max x y))))
crazy x y = floor $ \negate \tan \sin \max x y$
crazy = floor . negate . tan . sin . max
A reduction function is a function which takes a list and reduces the elements in the list to a single value. For example, sum and product are reduction functions:

```
GHCi> sum [1..10]
55
GHCi> product [1..10]
3628800
```
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```

What if we had a generalized reduction function which took a function and applied it across a sequence to obtain a result? Something like this:

```
reduce(f, seq) = f(f(f(seq[0], seq[1]), seq[2]), ...)
```
Haskell has a function called \texttt{foldr}, which takes a function, an initial value, and a list, and applies the function to each element in the list, recursively calling \texttt{fold} for the right value.

\[
\text{foldr } f \ z \ [\ ] = z
\]
\[
\text{foldr } f \ z \ (x : xs) = f \ x \ (\text{foldr } f \ z \ xs)
\]
Haskell has a function called `foldl`, which takes a function, an initial value, and a list. It recurses immediately, making the new initial value the result of calling the function on the initial value and the current element.

\[
\text{foldl } f z \; \text{[]} = z \\
\text{foldl } f z \; (x:xs) = \text{foldl } f \; (f \; z \; x) \; xs
\]
Examples: Folding

-- sum using foldl
sum' = foldl (+) 0

-- sum using foldr
sum' = foldr (+) 0

-- product
product' = foldl (*) 1
With your new learning groups, take some time preparing for the quiz using whatever study mechanism you wish.

Topics covered:

- Pattern Matching and Recursion
- Let, Where, Case, Guards